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# on Ship Design and Economics

## by

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1- "Structural and Economical Consequences of Ship Deflection", Seminar on the Application of Science & Technology in Marine Transport, A.M.T.A., J. Res. and Consultation Centre, July. (Egypt-1974), Shama, M. A.,
2- "Optimizing Hull Steel Weight for Overall Economic Transportation", Marine Week, May 2, (UK-1975), Shama, M. A.,
3- "The Cost of Irrationality in Ship Structural Design", PRADS. Int. Conference on Practical Design in Shipbuilding, SNAJ, Tokyo Oct. (Japan-1977), Shama, M. A.,
4- "Computer Design of Ships", Bull. Collage of Engineering, Basra University, (Iraq-1977), Shama, M. A.,
5- " Economical Consequences of Irrational Structural Design of Ships", Bull. Of Collage of Eng., Basra University, Vol.2, No.1, March, (Iraq-1977), Shama, M. A.,
6- "On the Rationalization of ship Structural Design", Schiff und Hafen, March, (Germany-1979), Shama, M. A.
7- "ON the Economics of Safety Assurance" Dept. of Naval Architecture and Ocean Engineering, Glasgow University, (UK-1979) Shama, M. A.,
8- "CADSUCS, the Creative CASD for the Concept Design of Container Ships", AEJ, Dec. (Egypt-1995), Shama. M. A., Eliraki, A. M. Leheta, H. W. and Hafez, K. A.,
9- "On the CASD of Container Ship; State of the Art", AEJ, Dec., (Egypt-1995) Shama, M. A., Eliraki, A. M. Leheta, H. W. and Hafez, K. A.,
10- "Assessment of Uncertainties of the Sea Margin", Maritime Research Journal, Sept., (Egypt-1998), Shama. M. A., Moniem, A. F. A. and Eweda, S.,

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# On the Economics of Safety Assurance

by

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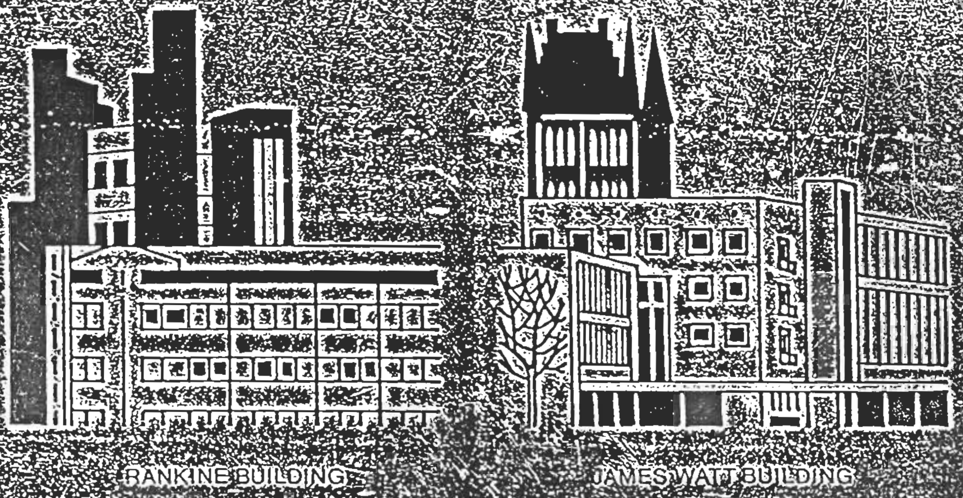

CONTENTS

# ON THE ECONOMICS OF SAFETY ASSURANCE

by

M.A. Shama, Ph.D.*

## Summary

The probabilistic approach to safety assurance of marine structures is examined. The emphasis is placed only on the failure resulting from the extreme values of load and strength. The various methods of calculating the probability of failure are given, together with a procedure suitable for non-normal density functions and illustrated by a numerical example. A method is presented for estimating the lower and upper limits of the probability of failure. Numerical values of these limits are also given for some specific cases.

The main factors affecting structural reliability are indicated. Particular emphasis is placed on the effect of truncation of the density functions of both loading and strength. The error resulting from using untruncated density functions is quantified. The effect of variability of structural strength, and reliability, with time is briefly considered.

The optimum structural reliability is examined. The optimality concept is based on the minimisation of the total cost, which includes both initial and operational costs.

The effect on the economy of transportation of reducing hull steel weight is stressed. It is shown that there is a wide scope for reducing steel weight of marine structures without reducing structural reliability. The importance of improving the design of local connections is emphasised.

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INTRODUCTION

The structural design of a ship affects her operation, safety and economical efficiency. The latter is generally influenced by hull steel weight, hull girder stiffness, complexity of constructional arrangements and the structural reliability of main hull girder and local connections. Therefore, ships should be designed and operated on a techno-economic basis with particular reference to fuel economy, reduced manning and maintenance costs and using adequate measures for quality assurance. A ship hull girder should not only have adequate strength to sustain the hostile sea environment, but should also have a low weight/strength ratio. The design process, therefore, should be based on realistic estimates of loading, powerful methods for computing hull girder and local stresses and deformations, rational design criteria for ensuring structural safety and a sound criterion for evaluating the economy of transportation (1,2).

The economy of transportation is improved by reducing initial and operational costs of a ship. Initial cost is influenced by hull weight, complexity of construction and the degree of structural reliability used in the design, among several other factors. Operational costs are also influenced by several factors, among them fuel consumption and structural reliability. Fuel costs represent nowadays a high percentage of the total operational costs. Adequate measures, therefore, should be taken to reduce fuel consumption. This could be achieved either by lowering service speed or by using the smallest and lightest ship for the required deadweight or capacity. Reducing hull steel weight could be achieved by structural optimisation techniques. These techniques, however, are normally directed towards improving weight/strength ratio of ship hull girder, without due regard to the impact on the economy of transportation. For deadweight carriers, hull weight reduction is a desirable requirement. Using optimum ship dimensions, it is possible to achieve significant weight saving in steel hull. This is confirmed by the reduction of about 600 tons of steel in a 47000 tons dwt tanker of the "pudgy type" (3). Also, a saving of 250 tons in a 211 m Ro/Ro was achieved merely by examining structural details. Using H.T.S. for the upper deck suggested a further saving of 500 tons. Much more saving could be achieved by rationalising the structural design process (4).

It/

It is essential to distinguish between structural optimisation of main hull girder and of local connections. The former involves efficient distribution of main scantlings to reduce weight/strength ratio without impairing structural reliability. The latter, however, may have adverse effect on the structural reliability of these connections by virtue of the presence of high stress gradients, stress concentration, notches, hard spots, complicated stress patterns, etc. The sensitivity of these local connections to geometrical and scantling variabilities may only produce marginal weight savings, but significant reduction in structural reliability. The deficiency of these local details may lead to local failures which, when accumulated and propagated, may induce serious structural failures. These failures increase maintenance and repair costs and the time out-of-service. Therefore, small cracks that may not immediately threaten ship structural safety, may subsequently have a deleterious effect on the economy of transportation. Consequently, structural safety and weight optimisation should be examined simultaneously.

It is evident now that the rationalisation of ship structural design should aim at:

i) reducing hull girder steel weight

ii) improving design of local structural connections and details

iii) identifying the possible modes of failure, for hull girder and local details, under various loading patterns and for different ship types

iv) developing methods for estimating structural reliability of both main hull girder and local connections

v) determining target safety levels based on the total economy of transportation

DEFINITIONS/

DEFINITIONS

a) Failure

Failure is the cessation of the performance of one function, or more, of a system, or member. It is a random event and is defined in terms of specified limit state or mode of failure. The modes of failure could be defined in terms of (4):

| | | | |
|---|---|---|---|
| i) | yielding | iv) | ultimate strength |
| ii) | buckling | v) | fatigue |
| iii) | excessive deformations | vi) | brittle fracture |

A pure failure mode represents a particular limit state reached under a particular loading pattern for a particular structural member. A compound failure mode results from a compound limit state, such as the local and overall buckling of a thin-walled member.

In the context of marine structures, the limit state, failure, could be divided into:

i) service failures; damage, cracks, local fractures, deformations, severe corrosion, etc.

ii) collapse; catastrophic failures such as buckling of deck structure, breaking ship in two, etc.

Damages may result from high stress concentration, fatigue, etc. and may be caused by corrosion, manufacturing imperfections, residual stresses, etc.

Collapse may result from buckling, excessive loads, deterioration of material, progressive damages to local details, etc. The collapse mode is generally started with a damage to a simple structural element, or connection, and then propagated into a large structural component. Most of the cracks and fractures are initiated at slots, cut-outs, bracket connections, etc.

In the rational approach to design, failure is defined by its probability of occurence under the prevailing conditions, i.e. $p_f$.

Causes of failure/

Causes of failure

Analysis of case histories suggests that the causes of failure can be grouped into the following main categories (4):

i) occurence of extreme values of load or strength beyond the margins introduced for safety

ii) errors in design assumptions, calculations, etc.

iii) errors in construction, erection and material properties

iv) accidental overloading due to collision, grounding, explosion, etc.

Structural design procedures are almost totally based on category (i), although failure due to this category represents only 10-20% of all cases (4). Therefore, striving for extremely low figures of probability of failure of this category may not improve the total probability of failure. This may call for a realistic outlook to the methods used for assessing structural reliability.

The assessment of the probability of failure due to (iii) and (iv) is rather difficult. Therefore, the total probability of failure could be assumed to result from (i) and (ii), and should be used only qualitatively for comparing different conditions or different designs.

b) Reliability

Reliability is defined as the capability of a member, or system, to perform its assigned functions under given environmental conditions. It could be also defined as the probability that the system, or element, does not fail during its projected service life, i.e. the probability of safe operation, or failureless operation, of the system.

If the demand on the system is D and its capability for a particular mode of failure is C, then structural reliability is given by:

$$p_s = \text{Probability } (C > D) = P(C > D)$$

$$= \int_{C>D} f_{C,D}(c,d)\, dc\, dd$$

It is evident that $p_f = 1 - p_s$.

Structural reliability is, therefore, a probabilistic characteristic of the system, or element, and should not be regarded as a free variable which can be optimised. It is not an absolute measure of safety and should be related to the economic and social consequences of failure.

c) Demand

The demand, D, normally refers to the maximum value of loading likely to occur over the expected service life of a ship. It is influenced by ship parameters, configuration, mass distribution, speed, heading, sea state, etc. The loading generally varies over a wide spectrum, but for practical and economical reasons, D could be represented by a truncated density function. The lower limit could be assumed zero and the upper limit, i.e. maximum loading, should be carefully estimated over the expected service life.

The variability of loading could be represented by the probability density function (p.d.f.) p(q), q being the loading. p(q) could be determined either for a short term or a long term extending over the expected service life of the ship under investigation. Both theoretical and experimental methods could be used for the determination of p(q).

The variability of the maximum loading, Q, could be represented by a p.d.f. p(Q), which could be derived from the long term distribution of q, using asympotic relations (see Fig. 1 ). The asympotic relations commonly used for the maximum values are given in Appendix (1).

d) Capability

Under a specified demand D, and a particular mode of failure, the capability C represents a limiting state beyond which the structure is expected to fail, to be damaged or collapse. The mode of failure depends entirely on the type of loading and structural geometry and configuration.

The variability of C results from the variabilities of the mechanical properties of the material, dimensional tolerances, fabrication defects, residual stresses, initial distortions, accuracy of stress analysis, errors in mathematical modelling, corrosion, wear and tear, etc.

Practically/

Practically, the capability does not extend from $-\infty$ to $+\infty$. Therefore, the p.d.f. of C should be represented by a truncated density function, whose lower and upper limits give the feasible range of variation. The lower limit represents the critical value regarding failure and therefore should be controlled so as to give an acceptable safety margin, or degree of risk. The upper limit represents the unnecessary extra strength, and hence extra steel weight, which may have adverse economical consequences. Therefore, adequate measures should be taken to ensure a narrow capability density function.

The variability of the minimum capability could be represented by a p.d.f. p(R), which could be obtained from the p.d.f. of capability, using asympotic relation (see Fig. 1 ). The asympotic relations commonly used for the minimum values are given in Appendix (1).

SAFETY ASSURANCE

A major requirement for any marine structure is to be reasonably safe, not to have catastrophic failure, nor to cause much trouble in service due to frequent minor failures.

Safety, in this context, is today concerned not only with the structure itself, but also with external damage that may result as a consequence of failure. Therefore, safety is not an absolute measure and should be related to the economic and social consequences.

The fundamental equation for safety assurance is given by:

$$R > Q$$

where R = resistance

Q = load

The margin of safety is given by:

$$M = R - Q > 0$$

The "safety factor" is given by:

$$\gamma = R/Q > 1.0$$

Classification societies remain the main authority responsible for the assurance of safety for ships and marine structures. The methods commonly used are:

i) quality control for material and construction

ii) quality control of maintenance by the regular surveys

iii) provision of corrosion margin to compensate material deterioration and ensure adequate strength

iv) control design by specifying procedures and constraints

For conventional ships, the classification society rules are based on long experience and gradual development. They ensure an acceptable safety level of the structural performance. Structural reliability is based on data collected from ships in service, such as damage statistics. These statistical data are usually condensed and then treated deterministically for developing rules for the structural design of ships. In this deterministic approach, safety assurance is based on the irrational concept of the "safety factor". Because of the inconsistency and lack of uniformity of these safety factors, marine structures designed according to these rules are generally overdesigned. For unconventional marine structures, such as offshore structures, the extrapolation from available data may be extremely difficult. Therefore, a rational procedure is required for assessing and checking structural safety. In this rational approach, safety assurance should be based on the statistical parameters of both loading and strength. It is evident that neither the load Q nor the strength R can be represented by a single value. Both are functions of several random variables and can be only treated statistically. However, this probabilistic approach could be divided into two methods (5):

i) <u>Full-statistical method</u> (normally called level-3 method)

In this method, safety assurance is based on a complete probabilistic analysis of the whole structural system, or elements. The full probabilistic information of both load and resistance is required, together with the target failure probabilities.

ii) Semi-statistical method

This method is generally divided into two levels:

a - Safety index approach (level-2 method)
Structural safety is ensured by a safety index compatible with acceptable probability of failure

b - Partial safety factor approach (level-1 method)
Structural safety is ensured by a number of partial safety factors, normally three factors, taking account of the variation of maximum loading and minimum strength.

In the following analysis, these three methods are considered in more detail.

## 1 - Full-statistical method

This method is based on the estimation of the probability of failure, $p_f$, or the risk, for the particular mode of failure under investigation, using the p.d.f. of both load and resistance of the whole structure, or any part of it.

If the demand Q and the capability R depend on the random variables $X_i$, $i = 1,2 \ldots n$, $p_f$ could be evaluated as follows:

$$p_f = \int_{R<Q} p(x_1) \cdot p(x_2) \ldots\ldots p(X_n)\, dx_1 dx_2 \ldots\ldots dx_n$$

For the present state of affairs, it is economically unjustificable and technologically unfeasible to determine "exactly" the p.d.f's of both Q and R, for the whole structure and for various modes of failure. Therefore, for practical applications, the full probabilistic method may be used only for one element of the structure and also for one particular mode of failure. In this case $p_f$ is given by:

$$p_f = \int_{-\infty}^{\infty} p_{R,Q}\{r, (m-r)\}\, dr$$

Assuming/

Assuming that R and Q are statistically independent, $p_f$ is given by:

$$p_f = \int_0^{\infty} p(q)\{\int_0^{q} p(r)\,dr\}dq$$

$$= \int_0^{\infty} p(r)\{\int_r^{\infty} p(q)\,dq\}dr$$

where: $p(q)$ = p.d.f. of Q

$p(r)$ = p.d.f. of R

let $F_U(x) = P(U < x) = \int_0^{x} p(u)\,du$ , $U = R, Q$

Then $P_Q(x) = \int_x^{\infty} p(q)\,dq = 1 - F_Q(x)$

and $p_f = \int_{-\infty}^{\infty} F_R(q) \cdot p(q)\,dq$

$$= \int_{-\infty}^{\infty} \{1 - F_Q(r)\}p(r)\,dr$$

The probability of failure, $p_f$, is in fact the shaded area shown in Fig. 2. It is not difficult to recognise that $p_f$ has lower and upper limits, i.e.

$$p_{f_L} < p_f < p_{f_U}$$

The calculation of these limits is given in Appendix (2). The calculation of $p_f$ could also be carried out as follows:

Let $dV = p(r)\,dr$ and $U = \int_r^{\infty} p(q)\,dq$

Then $p_f \doteq \int_0^{1} U\,dV$

In this case, $p_f$ is the area under the curve shown in Fig. (3). A numerical procedure using this definition of $p_f$ is given in Appendix (3).

An alternative approach for calculating $p_f$ could be based on the distribution function of the margin of safety M. It is important to recognise that, regardless of the distributions of the individual variates, it is the distribution of M that is important in the calculation of $p_f$. In this $p_f$ is given by (see Fig. 2):

$$p_f = P(R < Q) = P(M < 0) = \int_{-\infty}^{0} p(m(dm$$

It should be realised that the probabilities of failure are very small and that the assertion "the probability of failure" equal "x" is meaningless in itself. The probability of failure acquire meaning only as a relative characteristic for estimating the structural reliability of various arrangements of the same structure, or of the same structure under different loading conditions.

Furthermore, if the failure modes are completely correlated, the probability of failure for each mode should approach the overall target risk. If the failure modes are independent, the total probability of failure is the sum of the individual probabilities. It is necessary here to recognise the difference between the probability of failure due to collapse $p_{f_c}$ and the probability of failure due to damage $p_{f_d}$ as shown in Fig. (2). It is evident that:

$$p_{f_c} \ll p_{f_d}$$

Estimation of the required mean resistance

Using the target risk value $p_f$ and the p.d.f.'s of both R and Q, it would be possible, for some special cases, to determine the required mean value of resistance. Consider the following two cases (6):

(a) R and Q are both normally distributed

If $X \equiv N(\bar{x}, \sigma_x)$ , $X = R, Q$

Then $p_f = 1 - \Phi\left(\frac{\bar{r} - \bar{q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}}\right)$

Where: $\Phi(x)$ = the tabulated cumulative probability of the standard normal variate X for $X \leq x$

Hence, for a specified value of $p_f$, the required mean resistance is given by:

$$\bar{r} \geq \bar{q} + \Phi(1 - p_f)\sqrt{\sigma_R^2 + \sigma_Q^2}$$

(b)/

(b) R and Q are both log-normally distributed

If the coefficients of variation of R and Q are $v_R$ and $v_Q$ respectively, and $v_X \leq 0.3$, X = R,Q, $p_f$ is given by:

$$p_f \simeq 1 - \Phi\left[\frac{\ln(\frac{\bar{r}}{\bar{q}})}{\sqrt{v_R^2 + v_Q^2}}\right]$$

The required mean resistance for a specified value of $p_f$ is given by:

$$\bar{r} \geq \bar{q} \exp\{\Phi^{-1}(1 - p_f)\sqrt{v_R^2 + v_Q^2}\}$$

2 - The safety index approach

It is evident that it is technologically and economically unrealistic to totally eliminate the uncertainties associated with the parameters involved in the determination of both R and Q. It is also unfeasible to determine exactly the p.d.f.'s of both R and Q. Consequently, some practical procedures must be developed and introduced so as to be used for safety assurance.

In the safety index approach, the full statistical information of load and resistance are not required. Only the statistical parameters of the distribution functions of R and Q are required (the mean and standard deviation)

Assuming that R and Q are statistically independent, the mean and variance of the margin of safety M could be derived from the mean and variance of both R and Q as follows:

$$M = R - Q$$

$$\bar{m} = \bar{r} - \bar{q}$$

$$\sigma_M^2 = \sigma_R^2 + \sigma_Q^2$$

The safety index β is defined as the number of standard deviations of M between its mean value and zero, see Fig. 4:

$$\beta = \frac{\bar{m}}{\sigma_M} = \frac{\bar{r} - \bar{q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

Therefore, for a specified value of the safety index, $\beta_o$, the mean resistance should satisfy the following relation:

$$\bar{r} \geq \bar{q} + \beta_o \sqrt{\sigma_R^2 + \sigma_Q^2}$$

Introducing the approximation:

$$\sqrt{\sigma_R^2 + \sigma_Q^2} \simeq 0.75(\sigma_R + \sigma_Q)$$

The required mean resistance is given by:

$$\bar{r} \geq \bar{q} + 0.75\ \beta_o(\sigma_R + \sigma_Q)$$

The target safety index $\beta_o$ varies with loading (static, dynamic, etc.), with type of failure mode (damage, collapse, etc.) and with type of material.

The safety index $\beta$ could be also given in terms of the coefficients of variation of R and Q and the central safety factor as follows:

$$\beta = \frac{\theta - 1}{\sqrt{\theta^2 v_R^2 + v_Q^2}}$$

where $\theta = \bar{r}/\bar{q}$ , $v_R = \sigma_R/\bar{r}$ , $v_Q = \sigma_Q/\bar{q}$

Relationship between $\beta$ and $p_f$

The probability of failure $p_f$ could be estimated for a particular value of the safety index $\beta$, when M is assumed to follow the normal p.d.f., as follows:

$$M \equiv N(\bar{m}, \sigma_M)$$

$$p_f = \int_{-\infty}^{0} p(m)\,dm = \frac{1}{\sigma_M\sqrt{2\pi}}\ e^{\frac{-(m-\bar{m})^2}{2\sigma_M^2}}\ dm$$

Using the transformation : $t = \dfrac{m-\bar{m}}{\sigma_M}$

$$p_f = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\beta} \exp(-\frac{t^2}{2})\,dt = \Phi(-\beta)$$

Where: $\Phi(x)$ = cumulative probability of the standardised normal variate X.

The sensitivity of estimating $p_f$ in terms of $\beta$ could be realised from the following table:

| $\beta$ | 3.0 | 3.1 | 3.15 | 4.0 | 4.15 |
|---|---|---|---|---|---|
| $p_f$ | $1.35 \times 10^{-3}$ | $0.97 \times 10^{-3}$ | $0.815 \times 10^{-3}$ | $3.2 \times 10^{-5}$ | $1.5 \times 10^{-5}$ |

If, however, R and Q are not statistically independent, the variance of M is given by:

$$\sigma_M^2 = \sigma_R^2 + \sigma_Q^2 - 2\,\delta\,\sigma_R\sigma_Q$$

Where: $\delta$ = coefficient of correlation.

In this case, the safety index $\beta$ is given by:

$$\beta = \frac{\bar{r} - \bar{q}}{\sqrt{\sigma_R^2 + \sigma_Q^2 - 2\delta\sigma_R\sigma_Q}} = \frac{\theta - 1}{\sqrt{\theta^2 v_R^2 + v_Q^2 - 2\delta\theta v_R v_Q}}$$

It is evident that the calculation of $\beta$ depends on the estimation of $\bar{r}$, $\bar{q}$, $\sigma_R$ and $\sigma_Q$. In the general case, the random variables R and Q are non-linear functions of several other parameters:

$$R = R(x_1, x_2, \ldots\ldots\ldots x_n)$$

$$Q = Q(y_1, y_2, \ldots\ldots\ldots y_n)$$

In this case, R and Q may be linearised by expanding their functions in a power series in the neighbourhood of their mean values and neglecting the non-linear terms. The mean and variance can then be approximated by:

$$\bar{r} = R(\bar{x}_1, \bar{x}_2, \ldots\ldots \bar{x}_n)$$

$$\bar{q} = Q(\bar{y}_1, \bar{y}_2, \ldots\ldots \bar{y}_m)$$

$$\sigma_R^2 = \sum_{i=1}^{n} \left(\frac{\partial R}{\partial x_i}\right)^2 \sigma_{x_i}^2$$

$$\sigma_Q^2 = \sum_{j=1}^{m} \left(\frac{\partial Q}{\partial y_j}\right)^2 \sigma_{y_j}^2$$

## 3. The partial safety factor approach

In this approach the fundamental equation of the method of limit state design is given by (5):

$$R_k \geq \gamma Q_k$$

or $$R_k/Q_k \geq \gamma_f \cdot \gamma_c \cdot \gamma_m$$

where: $R_k$ = characteristic value of R and is defined as (see Fig. 5):

$$R_k = \bar{r}(1 - k_R v_R)$$

$Q_k$ = characteristic value of Q and is given by:

$$Q_k = \bar{q}(1 + k_Q v_Q)$$

$\gamma$ = overall safety factor

$\gamma_f$, $\gamma_c$, $\gamma_m$ = partial safety factor components, $\gamma_f$ and $\gamma_c$ take account of the variability of loading and structure modelling, $\gamma_m$ takes account of variability of material.

$k_R$ and $k_Q$ = specified constants selected to ensure that $R < R_k$ and $Q > Q_k$ are rare events.

In this approach, the objective uncertainties are taken care of by means of the characteristic loads and strengths, while the subjective uncertainties are covered by the partial safety factors.

The characteristic values of load and strength could be estimated from their corresponding p.d.f.'s as follows (see Fig. 9):

$$P(R < R_k) \leq p$$

$$P(Q > Q_k) \leq q$$

where p and q are specified small values.

In this approach, structural safety is ensured by the proper selection of p, q, $\gamma_f$, $\gamma_c$ and $\gamma_m$.

The fundamental equation of the limit state design could be simply stated as follows:

$$\gamma_R \bar{r} \geq \gamma_Q \bar{q}$$

where $\gamma_R$ = strength factor , $\gamma_R \leq 1.0$

$\gamma_Q$ = load factor , $\gamma_Q \geq 1.0$

Using the approximation (6):

$$\sqrt{\sigma_R^2 + \sigma_Q^2} \simeq 0.75(\sigma_R + \sigma_Q)$$

the strength and load factors could be given in terms of the safety index and the coefficients of variation of R and Q as follows:

$$\gamma_R \simeq \frac{1 - 0.75\beta v_R}{1 - k_R v_R}$$

$$\gamma_Q \simeq \frac{1 - 0.75\beta v_Q}{1 + k_Q v_Q}$$

If both R and Q are log-normally distributed, their characteristic values are given by (6):

$$R_k = \bar{r} \exp(- k_R v_R)$$

$$Q_k = \bar{q} \exp(k_Q v_Q)$$

The corresponding strength and load factors:

$$\gamma_R = \exp -\{(0.75\beta - k_R) v_R\}$$

$$\gamma_Q = \exp \{(0.75\beta - k_Q) v_Q\}$$

Factors affecting the probability of failure

The probability of failure is a characteristic of the structural system, or an element of this system, for the particular mode of failure and under the particular loading system. Its magnitude is influenced by the following factors:

i) The deterioration of structural capability with time as a result of corrosion, damage accumulation, etc. Therefore, the ability of a structure to maintain its original level of structural efficiency over its service life can be significantly improved by corrosion control, minimisation of surface area, provision of good drainage, access for cleaning, access for inspection, painting, etc.

ii) The assumed density functions for both resistance and loading. Since $p_f$ depends mainly on the shape of the upper tail of the loading function and the lower tail of the resistance function, its magnitude will be very sensitive to the type of functions assumed.

iii) Degree of truncation

Structural capability cannot physically vary from $-\infty$ to $+\infty$ or even attain zero value (7). Therefore, the p.d.f. of R should be truncated on both sides of the density curve. Similarly, the loading cannot attain infinite values, and therefore, the p.d.f. of Q should be also truncated, at least at its upper tail.

The truncated density function could be derived from the assumed theoret: p.d.f. as follows:

Let $p_X(x)$ = p.d.f. of X , $-\infty \leq X \leq \infty$

$f_X(x)$ = p.d.f. after truncation, $x_l \leq X \leq x_u$

$x_l, x_u$ = lower and upper feasible limits of the random variable X.

Then the truncated density function is given by (8):

$$f_X(x) = p_X(x)/H$$

and the cumulative distribution function is given by:

$$F(x) = \frac{P(x) - P(x_l)}{H}$$

where:
$$P(x) = \int_{-\infty}^{x} p_X(x)\,dx$$

$$H = \int_{x_l}^{x_u} p_X(x)\,dx = P(x_u) - P(x_l)$$

If $0 \leq X \leq \infty$, and the truncation is required only in the upper tail of the density function, then:

$$F(x) = \frac{P(x)}{H}$$

$$H = P(x_u) = \int_{0}^{x_u} p_X(x)\,dx$$

In the calculation of $p_f$, it is not necessary to use truncated density functions except when the actual density function is highly truncated. The error resulting from using a theoretical density function and not a truncated density function is given in Appendix (4).

It should be realised that it is possible to remove or reduce the extreme loads using suitable control measure (9), the resulting truncation of the demand distribution could lead to a reduction in required strength without affecting the structural reliability. Control of loading could be achieved by putting restrictions on the magnitude and distribution of the SWBM, and by using devices for monitoring dynamic stresses at sea. Similar results could be obtained by controlling the factors impairing structural capability. This could be achieved by improving material properties, controlling production processes, improving structural maintenance, etc.

The obvious result of these truncations is the possibility of reducing hull steel weight, by using reduced scantlings, without reducing structural safety.

Design for safety and economy

The rationalisation of ship structural design should aim at reducing hull steel weight, operational costs (fuel, maintenance, repair, etc.), increasing structural reliability and economical efficiency (2).

The/

The designer has to decide upon the best compromise between the desirable qualities of least weight, least cost and least unreliability. Cost in this context implies building and operational costs. Concern for keeping building and operational costs at the lowest possible level requires structures which are economic to build, operate and maintain.

Minimisation of building costs could be achieved by the rational selection of permissible imperfections and tolerances with due regard to both production costs and structural reliability. Production costs could be divided into:

i) costs of main hull items, such as panels, girders, etc.

ii) costs of connections, joints, cut-outs, etc. such as brackets, lugs, etc.

Poor design of connections and joints reduces their reliability considerably and promotes failures (10). Therefore, improving the design of these connections may lead to raising stress levels of structures clear of these joints without affecting the overall capability and structural reliability. This will evidently lead to saving in overall weight and costs.

Because of the increased concern for operational and out-of-service costs, reliability, serviceability and habitability require special attention to reduce in-service operating costs. Hence, structures should be designed with due regard to economy in operation, with particular reference to inspection, maintenance, repair, etc.

The failure of different parts of a marine structure has usually quite different consequences in terms of structural performance or economical considerations. Structural failures result mainly from fatigue cracks, brittle fracture, corrosion, wear, physical damage due to mishandling or accidents, grounding, collision, etc. The annual cost of damage repairs depends on the quality of design and construction, particularly for structural detials. Therefore, it is expected that there is an optimal level of reliability representing the best compromise between initial and maintenance costs of marine structures.

Standard reliability (safety level)

For any project or system, the benefits should exceed the penalties, or costs, i.e.

$$\text{Benefits} > \text{Costs}$$

$$\text{or} \quad \text{utility} = \text{benefits} - \text{costs}$$

The selection of a design should be based on the achievement of maximum utility and minimum expected loss in case of failure. The optimality concept includes minimum weight, minimum cost, high utilisation, high reliability, long service life,etc. The achievement of all these desirable features at the same time is rather impossible. This could be practically realised by the minimisation of the expected loss associated with failure, while imposing certain limiting conditions on the utility, or benefits.

Since intital and operational costs are both functions of reliability, the determination of the standard reliability (or safety level) is a techno-economic problem. As structural capability and demand are both stochastic phenomena,structural reliability is also a stochastic phenomenon. The determination of the standard reliability without taking into account the time factor does not give a rational solution. Structural reliability reduces with time by virtue of corrosion and damage accumulation (fatigue, cracks, bumps, etc.). These deteriorating effects can be catered for by considering structural capability as a random variable with a mean that is a decreasing function of time. The mean of the extreme loading is also a random variable that is an increasing function of time, as shown in Fig. 6. Therefore, the standard reliability, or safety level, should be determined from economical considerations (minimisation of cost of failure) for the assumed service life.

The standard reliability, safety level or the target total probability of failure, $p_f$, may be obtained from:

i) examination of comparative annual probabilities of failure in related activities. Based on the consequencies of failure, the following target annual failure probabilities are suggested, but have also been criticised (4):

normal/

i) (cont'd)

normal consequences : $p_f = 1 \times 10^{-5}$

serious consequences : $= 1 \times 10^{-6} - 1 \times 10^{-7}$

slight consequences : $= 1 \times 10^{-3} - 1 \times 10^{-5}$

It is also suggested that for important structures:

$$p_f \leq 3 \times 10^{-6}.$$

ii) considering the chances of death, due to failure, of persons using or associated with the structure concerned, target level of total annual risk, $p_f = 10^{-4} \cdot \frac{K_s}{n}$

where: $n$ = number of persons involved in the accident
$K_s$ = social criterion factor reflecting the activity associated with the structure ($K_s = 5$ for hazardous operations, such as marine structures).

This proposal has also been badly criticised.

iii) using economic criteria, particularly when loss of life is not involved, and examining the consequences of failure for the purpose of minimising these consequences.

A marine structure of low reliability has a short service life, rapidly goes out of service and requires large expenditures for maintenance and repair. An increase in reliability implies a rise in the intial cost and possible reduction in operational costs. Therefore, the choice of the optimum safety level should differ between owners and builders.

Determination of the reliability standard (optimum $p_f$)

The optimum $p_f$ is determined from the minimisation of the total cost. The latter could be divided into:

i)/

i) Non-failure cost items

- initial cost
- scrap value
- depreciation
- insurance
- maintenance

ii) Failure cost items

- replacement cost
- cost of repair
- cargo loss
- salvage cost
- loss due to time out-of-service
- temporary charter of replacement ship
- cost of pollution abatement, clean up, or other environmental effect
- loss of reputation, business and public confidence

Some of these cost items are independent of $p_f$ while others are totally dependent on it.

In order to simplify the analysis, the generalised cost equation is given by (11):

total cost (C) = initial construction cost ($C_I$) + present worth of the expected cost associated with failure ($C_F$)

The present worth PW of a future sum F is given by (12):

$$PW = \Phi F$$

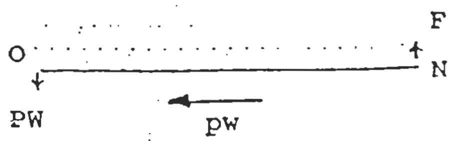
where:

$\Phi$ = $(PW - i\% - N)$

PW = present worth factor

i = rate of interest

N = number of compounding periods.



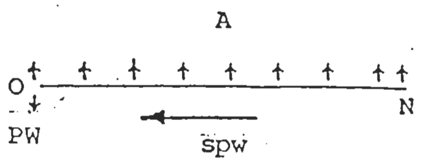

Also/

Also, the present worth PW of future annual payments A is given by:

$$PW = \Phi A$$

where: $\Phi$ = (spw - i% - N)

spw = series present worth factor




Hence, the generalised cost equation is given by:

$$C = C_I + \{p_f \cdot C_F\} \Phi$$

where: $\Phi$ = factor for transferring future cost of failure to its present worth value.

Optimality criterion

It is evident that $C_I$ increases with increasing structural reliability, i.e. with reducing $p_f$, whereas $C_F$ increases with increasing $p_f$. It is necessary, therefore, to determine the optimum value of $p_f$, which minimises the expected total cost.

Because of the scarcity of information regarding the economic consequences of failure, the criterion for selecting an optimal design is therefore based on the determination of the optimum safety level , risk, probability of failure, safety index, total factor of safety, etc., which gives the minimum expected total cost C, as illustrated by Fig. 7.

Optimum $p_f$ (reliability standard)

1) When $p_f$ is time dependent

The actual reliability of a marine structure P, is some function of the time t. Different longevity distributions may correspond to identical values of the initial and final reliabilities, P(O) and P(T) respectively, as shown in Fig. 8 (13).

The/

The structure following curve (1) is generally more reliable than the structure following curve (2). The selection of the function P(t) is based on the minimisation of the expected total cost C:

$$C = C_I + C_F \cdot \Phi$$

where $C_I$ and $C_F$ are both functions of the reliability P(t). The form of these functions depends on:

i) the type of economic model used

ii) type of marine structure, structural configuration, etc.

iii) type of loading

iv) properties of the material

v) type of mode of failure expected

vi) expected service life

It is evident that as the longevity increases, $C_F$ diminishes because of the physical wear and tear as well as amortisation.

The mathematical expectation of the cost C is given by (13):

$$\bar{C} = C_I - \int_o^T C_F \cdot \Phi \cdot \frac{dP(t)}{dt} dt$$

$$= C_I + \int_o^T C_F \cdot \Phi \cdot p(t)dt$$

In order to illustrate the calculation of $C_F\Phi$, assume that:

i) $p_f(t) = 1 - P(t) = 1 - e^{-p_o t}$

ii) $C_F\Phi = C_F e^{-(i-s)t}$

where: $p_o$ = annual failure probability
$i$ = rate of interest
$s$ = inflation rate

Hence/

Hence, $$C_F\Phi = \int_o^T C_F e^{-(i-s)t} \cdot p_o \cdot e^{-p_o t}\, dt$$

$$= C_F p_o \frac{1 - e^{(-i+s-p_o)T}}{i - s + p_o}$$

Thus, $$\bar{C} = C_I + C_F \cdot p_o \frac{1 - e^{(-i+s-p_o)T}}{i - s + p_o}$$

The function P(t) may be simply assumed as follows:

$$P(t) = 1 - \{1 - P(T)\}(\frac{t}{T})^2$$

$$= 1 - p_f(T)(\frac{t}{T})^2$$

where: $p_f(t)$ = probability of failure at t = T

Substituting in the generalised cost equation:

$$C = C_I + \int_o^T C_F \cdot \Phi \cdot p_f(T) \frac{2t}{T^2}\, dt$$

$$= C_I + \{C_F p_f(T)\}\Phi$$

However, in the general case, the generalised cost equation could be solved numerically as follows:

$$C_j \simeq C_{Ij} - \sum_{k=1}^{n} C_{Fj}(\frac{t_{k-1} + t_k}{2})(P_k - P_{k-1}) \cdot \Phi$$

where: $C_{Ij}$ and $C_{Fj}$ = values of $C_I$ and $C_F$ at reliability value $P_j$ corresponding to $t_j$, j = 1,2....n

$P_o$ = 1 , at t = o

ii) when $p_f$ is independent of time

Because of the lack of data on the variability of reliability with time and the complexity of calculations, it is sufficient at this stage to treat the structural reliability P as some number and ignoring the time factor and the/

the longevity concept. In this case, the reliability standard, optimum value of $p_f$, could be determined from the minimisation of the general cost equation:

$$C = C_I + \{C_F(1 - P)\}\Phi$$

$$= C_I + \{C_F \cdot p_f\}\Phi$$

where: $C_I$ and $C_F$ are both functions of P, or $p_f$.

The optimum $p_f$ is determined from the condition that (see Fig. 7):

$$\frac{\partial C}{\partial p_f} = 0$$

$$\text{i.e.} \quad \frac{\partial C_I}{\partial p_f} + \{p_f \frac{\partial C_F}{\partial p_f} + C_F\}\Phi = 0 \text{ ............ (a)}$$

Assuming that $C_I = A_o(1 - a \log_e p_f)$

and $C_F = C_o + m\,C_I$

where: $A_o$ = cost of that part of structure independent of $p_f$

a = a numerical factor

m = a factor representing the increase in cost of structure ($m \geq 1$)

Equation (a) gives:

$$\frac{1}{p_f} = \{\frac{C_o + mA_o}{aA_o} - m(1 + \log_e p_f)\}\Phi$$

$$\simeq \{\frac{C_o + mA_o}{aA_o}\}\Phi$$

Hence, $p_{f_o} \simeq 1/\{\frac{C_o + mA_o}{aA_o}\}\Phi$

where/

where: $p_{f_o}$ = optimum value of $p_f$

Therefore, the expected minimum total cost, $C_{min}$, is given by:

$$C_{min} \simeq C_I + \left[ \frac{C_F}{\{\frac{C_o + mA_o}{aA_o}\}} \right] \Phi$$

$$= C_I + \left[ \frac{C_F \cdot aA_o}{C_o + mA_o} \right] \Phi$$

If it is assumed that $C_F$ is independent of $p_f$, the optimum $p_f$ is given by:

$$p_{f_o} = \frac{(C_F)\Phi}{A_o a}$$

It should be realised that it is possible also to assume that $C_I$ and $C_F$ are both functions of the safety index $\beta$ or the total safety factor $\gamma$. It is evident that increasing the safety index $\beta$, or the total safety factor $\gamma$, reduces the probability of failure, increases the initial cost and reduces the cost of failure.

The determination of the optimum value of the safety index $\beta$ could be based on the following assumptions (14):

$$C_I = a(1 + b\beta)$$

$$C_F = mC_I$$

$$p_f = Ce^{-\beta/d}$$

For the normal p.d.f., $p_f = \Phi(-\beta)$

The generalised cost equation is given by:

$$C = a(1 + b\beta) + ma(1 + b\beta)Ce^{-\beta/d}$$

The/

The optimum value of $\beta$, $\beta_o$ should satisfy the following condition:

$$\frac{\partial C}{\partial \beta} = 0$$

i.e. $$e^{\beta/d} - \frac{mc}{d}\beta + \frac{mc}{bd}(bd-1) = 0$$

The solution of this equation gives, $\beta_o$, the optimum value of $\beta$, as shown in Fig. 9. Designing the structure for a value of $\beta \lessgtr \beta_o$ (such as $\beta_1$) decreases the utility of the structure.

CONCLUSIONS

The main conclusions drawn up from this investigation could be summarised as follows:

i) The statistical methods, theory of probability and reliability are very powerful and useful tools for the rationalisation of design of marine structures.

ii) The probability of failure, for any particular mode of failure, should be treated as a relative characteristic of the structural system and should be used only as a qualitative measure for comparing alternative designs or different conditions of the same design.

iii) The optimum standards of quality assurance could be determined from the minimisation of the total cost. However, because of the scarcity of information on the economic consequences of failure, the methods presented should be treated with caution and should be used only for qualitative purposes.

iv) Without reducing structural reliability, weight saving of hull steel/

iv) (Cont'd)

steel is possible and could be achieved by:

- truncating the density functions of either, or both, demand and capability, using control methods
- improving the design of local details and connections

v) The error in $p_f$ due to using untruncated density functions, for either or both load and strength, is insignificant, except when the actual density functions are highly truncated.

vi) Much work is needed in the following directions:

- developing methods to improve structural reliability of hull girder and local details
- application of the full-probabilistic approach to some specific problems where adequate data are available
- developing methods to control the extreme values of load (upper tail) and strength (lower tail)
- developing methods to determine the optimum values of structural reliability based on the total economy of transportation
- determination of structural reliability when both demand and capability are treated as stochastic phenomena
- collecting data on structural failures. These data should include costs of repair, period of repair, years of service, cause of failure , etc.
- developing methods of ensuring minimum values of structural reliability, such as failure-indication methods, structural testing and inspection, etc.

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# Appendix (1)

## EXTREME TYPE DISTRIBUTIONS

1. Distributions used for Maximum Values

The following extreme type distributions could be used (15):

a. Extreme Type I

i. Distribution function

$$F(x) = \exp\left\{-\exp(-\alpha\{x-u\})\right\} \qquad \alpha > 0$$

ii. Probability density function

$$f(x) = \alpha \exp\left\{-\alpha(x-u) - e^{-\alpha(x-u)}\right\}$$

iii. Mode $\tilde{x} = u$

iv. Mean $\bar{x} = u + \frac{0.57722}{\alpha}$

v. Standard deviation $\sigma = \pi/\sqrt{6} \cdot \alpha = \frac{1.2825}{\alpha}$

vi. Coefficient of variation $v_Q = \frac{1.2825}{\alpha u + 0.57722}$

b. Extreme Type II

i. Distribution function

$$F(x) = \exp\left\{-(Kx)^{-\beta}\right\} \qquad \beta > 0, \qquad K > 0, \qquad x \geq 0$$

ii. p.d.f. $f(x) = \beta K(Kx)^{-(\beta+1)} \exp\left\{-(Kx)^{-\beta}\right\}$

iii. Mode $\tilde{x} = \frac{1}{K}\left\{\frac{\beta}{\beta+1}\right\}^{1/\beta}$

iv. Mean $\bar{x} = \frac{1}{K}\Gamma\left(1 - \frac{1}{\beta}\right)$

v. Standard deviation $\sigma = \frac{1}{K}\left\{\Gamma\left(1 - \frac{2}{\beta}\right) - \Gamma^2\left(1 - \frac{1}{\beta}\right)\right\}^{\frac{1}{2}}$

vi. Coefficient of variation

$$v_Q = \left\{\frac{\Gamma(1-2/\beta)}{\Gamma^2\left(1 - \frac{1}{\beta}\right)} - 1\right\}^{\frac{1}{2}}$$

2. Distributions used for Minimum Values

The following extreme type distributions could be used to represent the minimum values of resistance, or capability:

a. Extreme Type I

i. Distribution function $F(x) = 1 - \exp\left\{- e^{\alpha(x-u)}\right\}$ $\alpha > 0$

ii. p.d.f. $f(x) = \alpha \exp\left\{\alpha(x-u) - e^{\alpha(x-u)}\right\}$

iii. Mode $\tilde{x} = u$

iv. Mean $\bar{x} = u - \frac{0.57722}{\alpha}$

v. Standard deviation $\sigma_R = \frac{1.2825}{\alpha}$

vi. Coefficient of variation $v_R = \frac{1.2826}{\alpha u - 0.57722}$

b. Extreme Type III (Weibull Distribution)

i. Distribution function

$$F(x) = 1 - \exp\left\{-\left(\frac{x-\varepsilon}{K-\varepsilon}\right)^{\beta}\right\} \qquad x \geq \varepsilon, \qquad \beta > 0, \qquad 0 \leq \varepsilon < K$$

ii. p.d.f. $f(x) = \frac{\beta}{K-\varepsilon}\left(\frac{x-\varepsilon}{K-\varepsilon}\right)^{\beta-1} \cdot \exp\left\{-\left(\frac{x-\varepsilon}{K-\varepsilon}\right)^{\beta}\right\}$

iii. Mode $\tilde{x} = \varepsilon + (K-\varepsilon)\left(1 - \frac{1}{\beta}\right)^{1/\beta}$ $\beta > 1$

iv. Mean $\bar{x} = \varepsilon + (k-\varepsilon)\ \Gamma\left(1 + \frac{1}{\beta}\right)$

v. Standard deviation

$$\sigma_R = (K-\varepsilon).\left\{\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right\}^{\frac{1}{2}}$$

vi. C.O.V. $v_R = \dfrac{(K-\varepsilon)\left\{\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right\}^{\frac{1}{2}}}{\varepsilon + (K-\varepsilon)\ \Gamma\left(1 + \frac{1}{\beta}\right)}$

Appendix (2)

## CALCULATION OF THE LOWER AND UPPER LIMITS OF $p_f$

When the p.d.f.s of both R and Q are known, it is possible to determine the domain of $p_f$, designated by $p_{f_L}$ and $p_{f_U}$,

$$p_{f_L} < p_f < p_{f_U}$$

where $p_{f_L}$ = lower limit of $p_f$

$p_{f_U}$ = upper limit of $p_f$

From Fig. (10), we have (13):

$$p_{f_L} = w_Q \cdot w_R$$

$$p_{f_U} = w_Q + w_R - w_Q \cdot w_R$$

where: $$w_Q = \int_{q_o}^{\infty} p(q)dq$$

$$w_R = \int_{o}^{r_o} p(r)dr$$

In some cases, it may be sufficient to estimate $p_{f_L}$ and avoid the laborious calculations of $p_f$.

In order to compare $p_{f_L}$ and $p_{f_U}$ with $p_f^*$ as derived from the safety index, it is assumed that both R and Q follow the normal density function, i.e.

$$X \equiv N(\bar{x}, \sigma_x) \quad , \qquad X = R, Q$$

Using coefficients of variation:

$$v_R = \frac{\sigma_R}{\bar{r}} \quad , \qquad v_Q = \frac{\sigma_Q}{\bar{q}}$$

and the central safety factor, $\theta = \bar{r}/\bar{q}$.

From Fig. (11): $r_o = \bar{r} - \alpha\sigma_R$

$$q_o = \bar{q} + k\sigma_Q = \bar{q}(1+kv_Q)$$

at $r_o = q_o$, we get

$$\alpha = \frac{\theta - 1 - kv_Q}{v_R \cdot \theta}$$

The following table shows the effects of variation of $\theta$, $v_R$, $v_Q$ and k on the magnitudes and range of variation of $p_{f_L}$ and $p_{f_U}$. The values of $p_f^*$ are calculated from the safety index $\beta$:

$$\beta \doteq \frac{\theta - 1}{\sqrt{\theta^2 v_R^2 + v_Q^2}}$$